QCD fits to the g_1 world data and uncertainties
THE HERMES EXPERIENCE

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TRIUMF/DESY

Global Analysis Workshop

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Outline

- *Definition of g_1*
- *Recent g_1 data from HERMES*
- *Fits to g_1 data*
- *Some ingredients in fits*
- *The χ^2*
- *Statistical uncertainties*
- *Systematic uncertainties*
- *Conclusions*

From the Measured Inclusive Asymmetries to g_1

$$g_1(x,Q^2) = \underbrace{\frac{1}{1-\frac{y}{2}-\frac{1}{4}y^2\gamma}}_{\text{kinematic factors}}\left[\underbrace{\frac{Q^4}{8\pi\alpha^2 y}\frac{\partial^2\sigma_{unpol}}{\partial x\partial Q^2}}_{\text{param.}}\underbrace{A_{||}(x,Q^2)}_{\text{measured}}+\underbrace{\frac{y}{2}\gamma^2}_{\text{kin. fact.}}\underbrace{g_2(x,Q^2)}_{\text{param.}}\right]$$

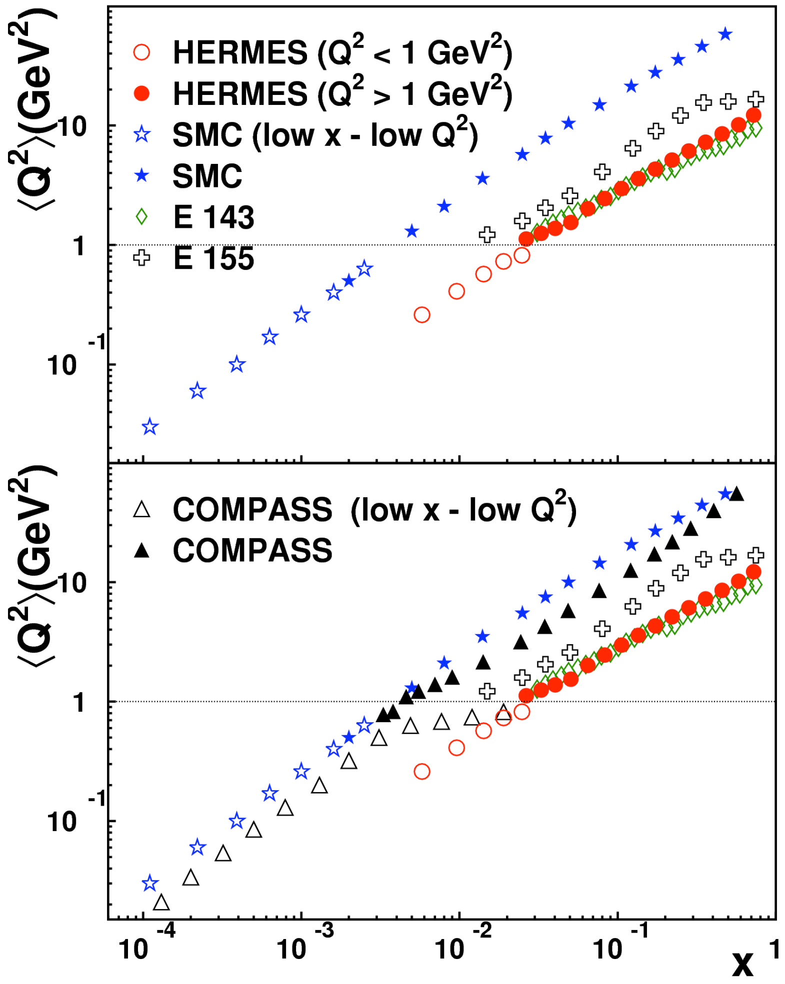
❖ *The measured quantity is the asymmetry*

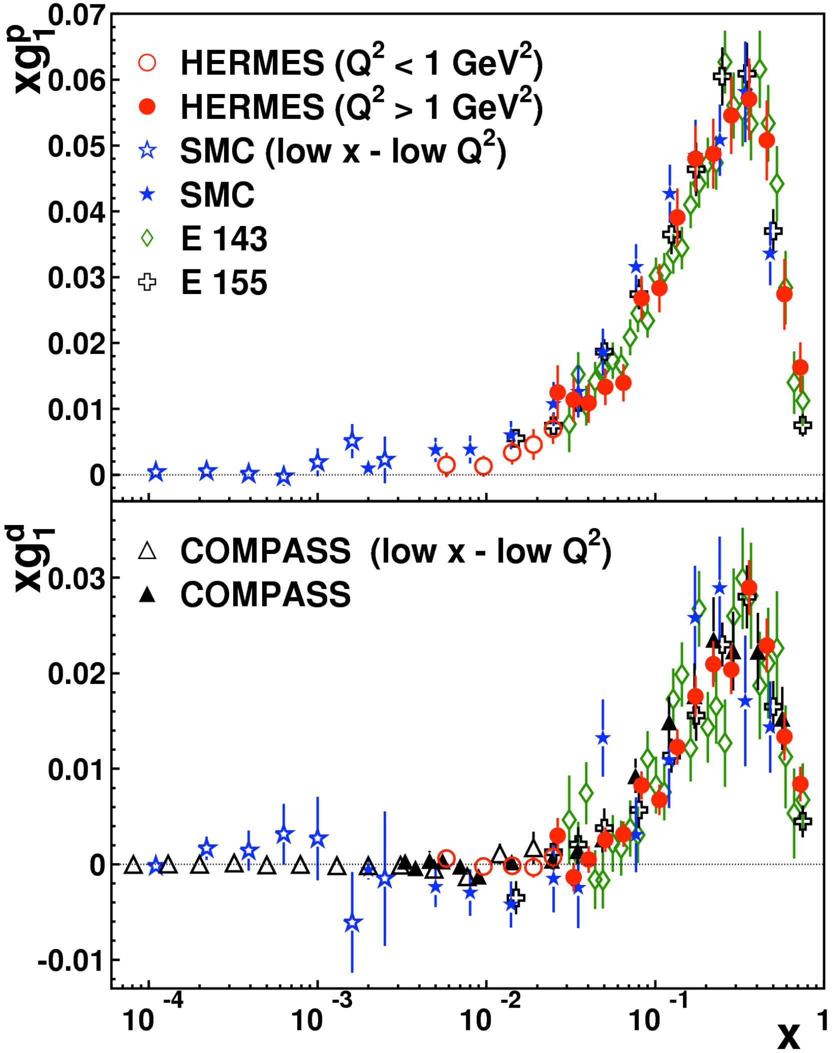
❖ *From the asymmetry, with the additional information on beam energy, σ_{unpol} and g_2 each experiment provides a value of g_1*

❖ *A first step in the analysis of world data consists in trying to make all data sets compatible wrt the choice of g_2 and σ_{unpol}*

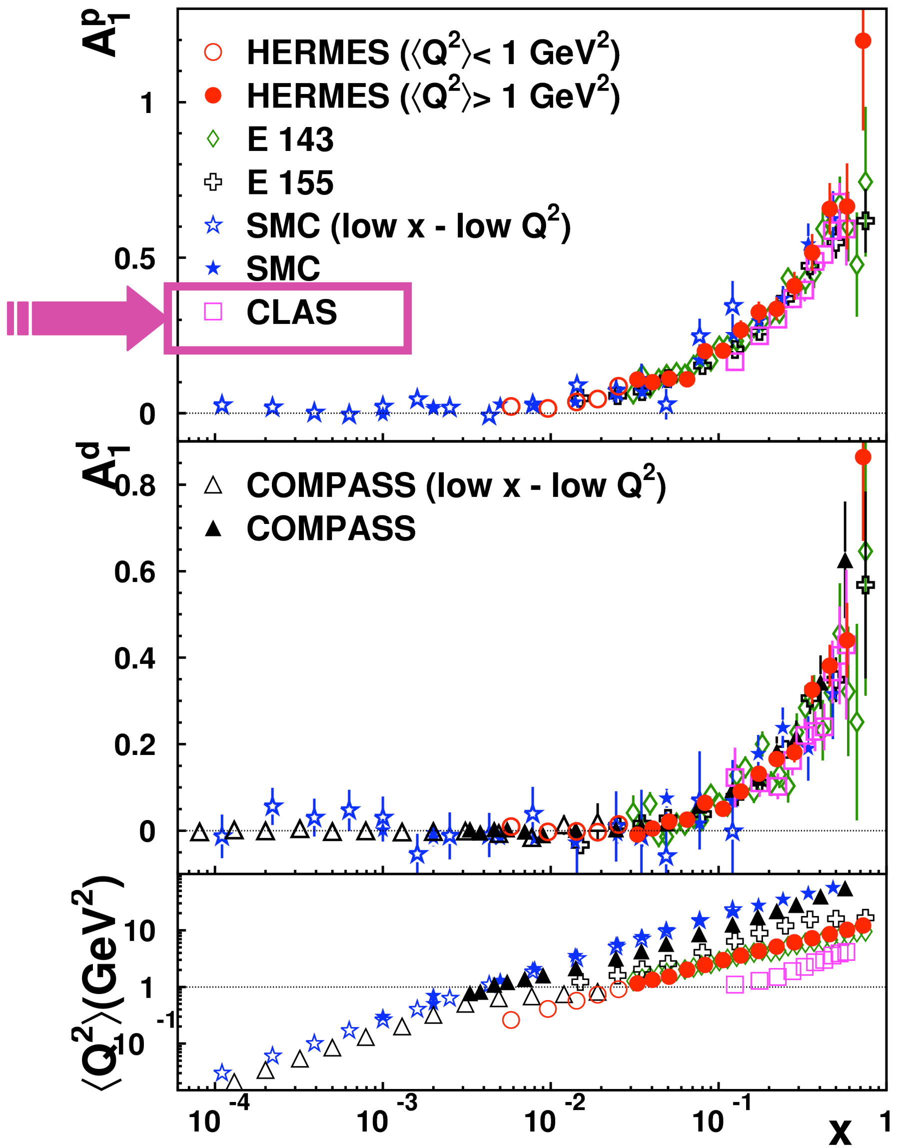
❖ *NB: σ_{unpol} depends on R and F_2, so the chosen F_2 has to be compatible with the chosen R*

g_1 World Data



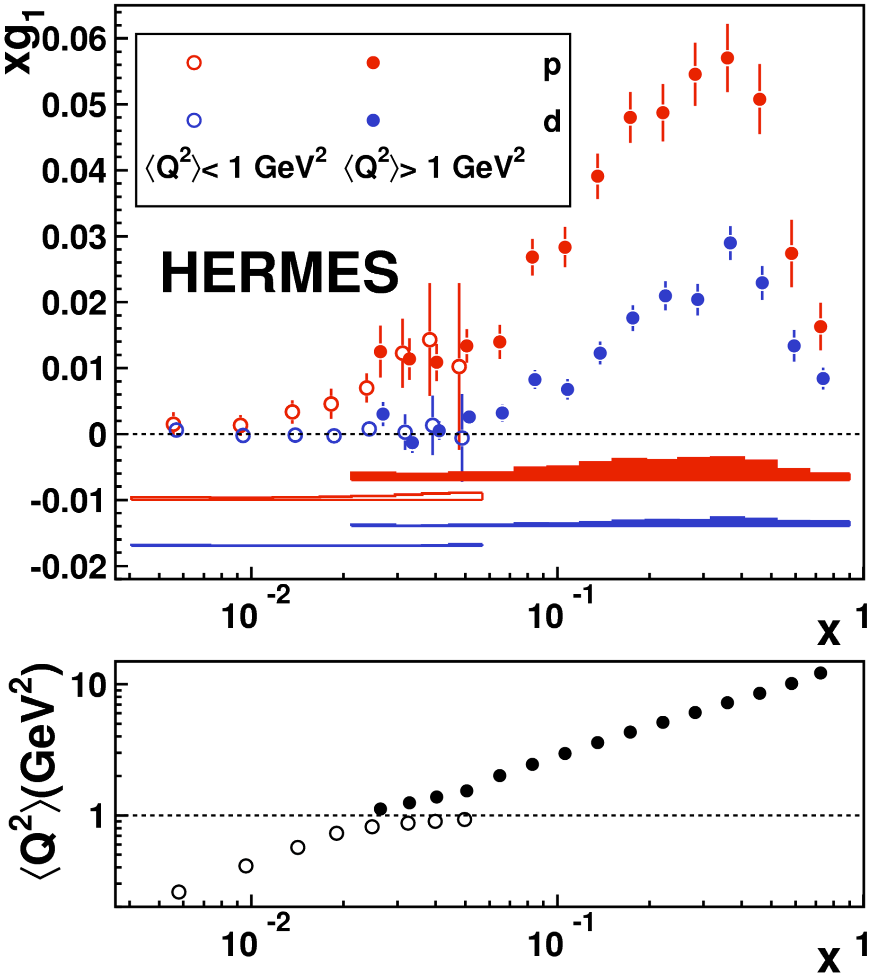


A_1 World Data



$$A_1 = \left(1+\gamma^2\right)\frac{g_1}{F_1} - \gamma A_2 \approx \frac{g_1}{F_1}$$

HERMES results

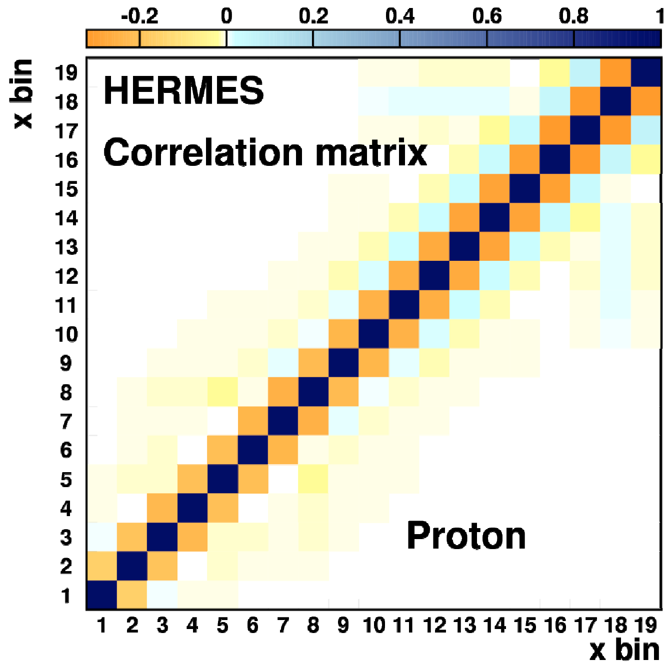


Phys.Rev.D75(2007)012007

0.0041 < x < 0.9

0.18 GeV² < Q² < 20 GeV²

- *Correction for smearing and radiative effects introduces statistical correlations*
- *Statistical uncertainties are diagonal elements of covariance matrix*



g_1 QCD fits

- g_1 QCD fits are models for $\Delta q(x,Q^2)$ obtained by fitting inclusive world data on g_1 :

$\Delta\Sigma$, Δq^p_{NS}, Δq^n_{NS}, ΔG
or
Δu_v, Δd_v, $\Delta\bar{q}$, ΔG
(need assumption on the sea)

Start from model at initial Q2=Q^2_0:

$$\Delta q_i(x,Q_0^2) = f_i(p_1^i,\ldots\ldots, p_{nq}^i)$$

DGLAP: $\dfrac{\partial \Delta q_i}{\partial Q^2} = \Delta q_i \otimes \Delta P_i$ to go from Q^2_0 to Q^2_{data}

Calculate g_1^{fit} at the Q^2 of all data points: $g_1^{fit}(x,Q^2) \approx \sum_i \left(\Delta q_i(x,Q^2) \otimes \Delta C_i\right)$

minimize $\chi^2 = \sum_{data} (g_1^{fit} - g_1^{data})_i \operatorname{cov}(i,j)(g_1^{fit} - g_1^{data})_j$
(for HERMES data)

get $p_1^1, \ldots\ldots, p_{n_1}^1, \ldots\ldots\ldots\ldots, p_1^{nq} , \ldots\ldots, p_{nq_n}^{nq}$

The ingredients of a fit

Many fits on the 'market' look at one or another problem, and it is sometimes very difficult to compare fits from different groups, as they are based on different assumptions.

- *Higher twists (LSS06)*
- *Inclusion of semi-inclusive data (dFS) or $A_{LL}^{\pi_0}$ (AAC)*
- *Statistical Uncertainties*
- *Systematic Uncertainties*

LSS'06

Initial parameterization:

$$\boxed{x\Delta f_i(x,Q_0^2) = \eta_i A_i x^{\alpha_i} x f^{MRST}(x,Q_0^2)}$$

$$\Delta f = \Delta u_v, \Delta d_v, \Delta s, \Delta G$$

New data:
- *Low Q^2 CLAS data*
- *COMPASS data (large Q^2)*

Higher twist terms included in the fit:

$$g_1(x,Q^2)_{HT} = \frac{h(x,Q^2)}{Q^2} + O\left(\frac{\Lambda^4}{Q^4}\right)$$

$$h^p(x_i), h^n(x_i)$$

i=1,…,5: 10 parameters

+6 for PDs (=8-2 for sum rules)

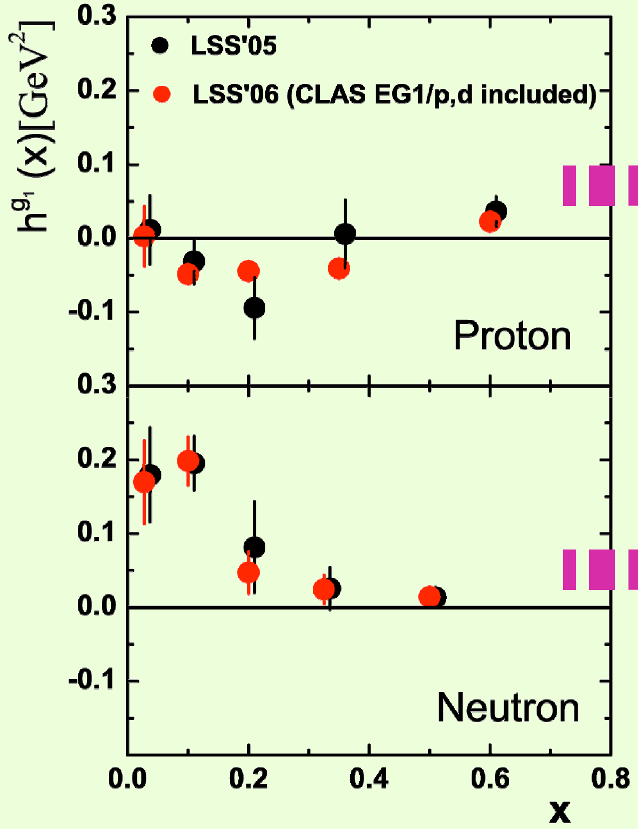
$$g_1(x,Q^2)_{LT} = g_1(x,Q^2)_{pQCD} + \frac{M^2}{Q^2} h_{TMC}(x,Q^2) + O\left(\frac{M^4}{Q^4}\right)$$

$$\left[\frac{g_1(x,Q^2)}{F_1(x,Q^2)}\right]_{\exp} \Leftrightarrow \frac{g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{HT}}{F_1(x,Q^2)_{\exp}}$$

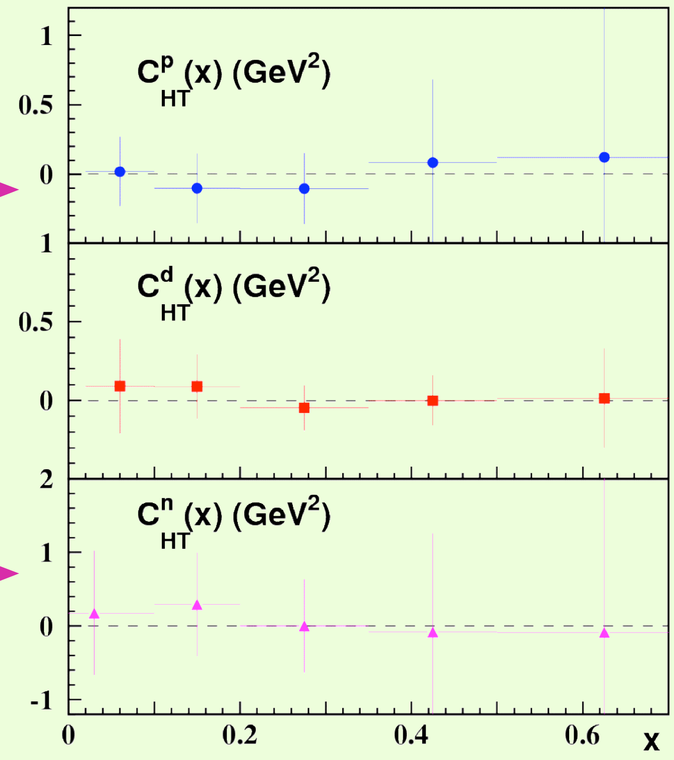
E.Leader et al.,Phys.Rev.D75(2007)074027

LSS'06: Higher Twists

LSS'06: Strong HT signal



Other groups (BB) ***claim*** *to have* ***not found*** *such a strong signal for HT*



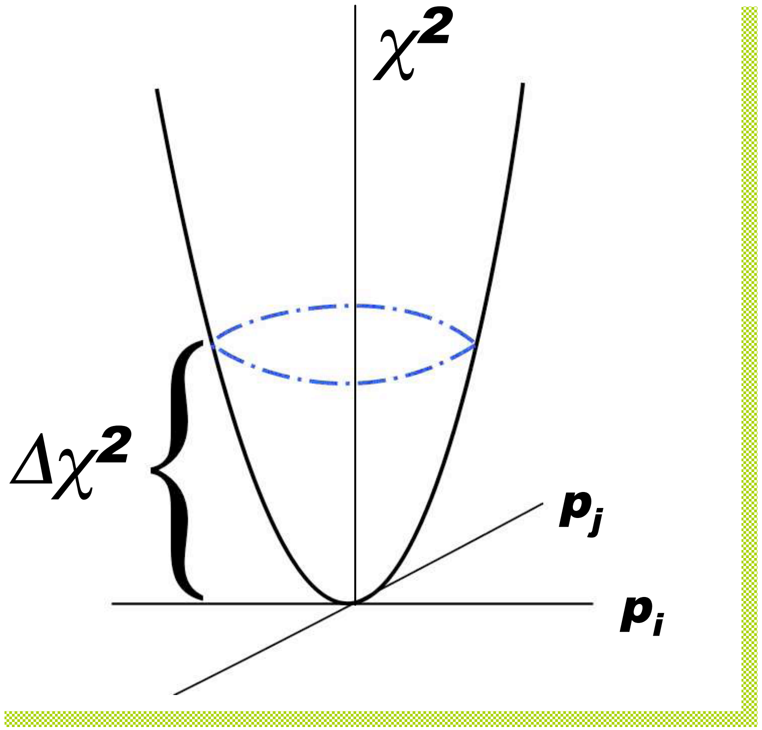
H.Boettcher, private comm.

- *The values are close, but the conclusions are different!*
- *Diff. depend on the size of the error bars and on their meaning*

Statistical Uncertainties

- At least two groups (BB and AAC) report statistical uncertainties inflated by $\approx \sqrt{NPAR}$
- They cannot be directly compared to those of other groups.

(These inflated uncertainties do not correspond to what is normally understood as statistical uncertainty, obtained as the ***standard deviation of the distribution of results*** *derived by fitting a large number of MC data sets resembling the experimental data sets, but with* ***each data point fluctuating independently*** *according to the experimental statistical uncertainty)*



- *$\Delta\chi^2=1$ defines the 1σ uncertainty for single parameters*

- *$\Delta\chi^2 \sim NPAR$ is the 1σ uncertainty for the NPAR parameters to be* ***simultaneously*** *located inside the hypercontour*
($\Delta\chi^2$.ne.1 normally used for ***unknown systematics****, see CTEQ, MRST….)*

NPAR=number of parameters

Evolution of Statistical Uncertainties

Statistical uncertainties are given by:

X-space

$$(\sigma_{\Delta q})^2(x,Q^2) = \sum_{ij} \frac{d\Delta q}{dp_i}(x,Q^2)\frac{d\Delta q}{dp_j}(x,Q^2)\,\mathrm{cov}(p_i,p_j)$$

❖ *Calculable exactly at Q^2_0 since the functional form of Δq is known at Q^2_0.*

$$\frac{d}{dp_k}$$

$$\frac{d}{dt}\Delta q_{NS} = P_1 \otimes \Delta q_{NS}$$
$$\frac{d}{dt}\Delta\Sigma = P_2 \otimes \Delta\Sigma + P_3 \otimes \Delta G$$
$$\frac{d}{dt}\Delta G = P_4 \otimes \Delta\Sigma + P_5 \otimes \Delta G$$

$$\Rightarrow$$

$$\frac{d}{dt}\frac{d\Delta\Sigma}{dp_{\Sigma_i}} = P_2 \otimes \frac{d\Delta\Sigma}{dp_{\Sigma_i}} + P_3 \otimes \frac{d\Delta G}{dp_{\Sigma_i}}$$
$$\frac{d}{dt}\frac{d\Delta G}{dp_{\Sigma_i}} = P_4 \otimes \frac{d\Delta\Sigma}{dp_{\Sigma_i}} + P_5 \otimes \frac{d\Delta G}{dp_{\Sigma_i}}$$

❖ *The derivatives of the distributions* ***evolve just like the distributi***

Evolution of Statistical Uncertainties

Parameter of
$\Delta\Sigma\,(x,Q^2_0)$

$$\frac{d}{dt}\frac{d\Delta\Sigma}{dp_{\Sigma_i}} = P_2 \otimes \frac{d\Delta\Sigma}{dp_{\Sigma_i}} + P_3 \otimes \cancel{\frac{d\Delta G}{dp_{\Sigma_i}}}$$

$$\frac{d}{dt}\frac{d\Delta G}{dp_{\Sigma_i}} = P_4 \otimes \frac{d\Delta\Sigma}{dp_{\Sigma_i}} + P_5 \otimes \cancel{\frac{d\Delta G}{dp_{\Sigma_i}}}$$

At initial $Q^2=Q^2_0$

- *Only at Q^2_0 ΔG does not depend on the $\Delta\Sigma$ parameters!*
- ***ΔG acquires a dependence on the $\Delta\Sigma$ parameters** in Q^2*
- *The same is true for $\Delta\Sigma$ (it depends on the ΔG parameters through the evolution)*
- *The **NS evolves independently** from the other distributions*
- *For details on the unc. calculations in Mellin space see BB paper (*Nucl.Phys.**B636**(2002)225)

Normalization Uncertainties

❖Account for the (substantial) syst. unc. common to an entire data set for one experiment by adding it incoherently in quadrature to the uncertainty in each data point.

❖Done with a χ^2 **penalty term,** see BB**:**

Fitted normalization

$$\chi^2 = \sum_{i=1}^{n^{\exp}} \left[\frac{(N_i - 1)^2}{(\Delta N_i)^2} + \sum_{j=1}^{n^{data}} \frac{\left(N_i g_{1,j}^{data} - g_{1,j}^{theor}\right)^2}{\left(N_i \Delta g_{1,j}^{data}\right)^2} \right]$$

Norm. unc. quoted by expt.

❖The normalizations can also be calculated ***analytically*** at each step**,** without increasing the number of parameters in the fit:

$$\left(\frac{\partial \chi^2}{\partial N_i} = 0\right) \Longrightarrow N_i = 1 + \frac{\sum_{k=1}^{n^{data}} g_{1,k}^{theor}\left(g_{1,k}^{theor} - g_{1,k}^{data}\right) / \left(\Delta g_{1,k}^{data}\right)^2}{\left(\frac{1}{\Delta N_i}\right)^2 + \sum_{j=1}^{n^{data}} \left(g_{1,j}^{theor}\right)^2 / \left(\Delta g_{1,k}^{data}\right)^2}$$

Systematic uncertainties

One can also consider the experimental systematic uncertainties:

$$g_{1,k}^{data} \Rightarrow g_{1,k}^{data} + S_i \cdot \Delta_{sys} g_{1,k}^{data}$$

***S**$_i$ can also be calculated analytically at each step*

$$\chi^2 = \sum_{i=1}^{n^{\exp}} \left[\sum_{j=1}^{n^{data}} \frac{\left(g_{1,j}^{data} + S_i \cdot \Delta_{sys} g_{1,j}^{data} - g_{1,j}^{theor}\right)^2}{\left(\Delta g_{1,j}^{data}\right)^2} + S_i^2 \right]$$

$$\left(\frac{\partial \chi^2}{\partial S_i} = 0\right) \Longrightarrow S_i = \frac{\sum_{k=1}^{n^{data}} \left(g_{1,k}^{theor} - g_{1,k}^{data}\right) \Delta_{sys} g_{1,k}^{theor} / \left(\Delta g_{1,k}^{data}\right)^2}{1 + \sum_{j=1}^{n^{data}} \left(\Delta_{sys} g_{1,j}^{theor}\right)^2 / \left(\Delta g_{1,j}^{data}\right)^2}$$

Putting it all together

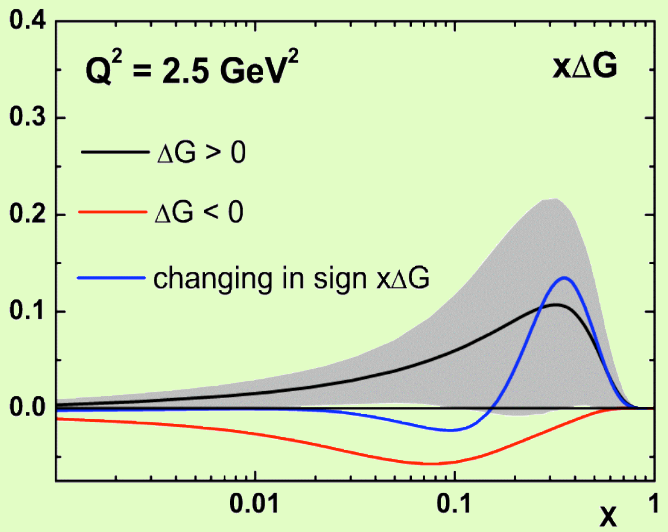
- We finally get the χ^2 including
  - Covariance (for HERMES data)
  - Normalizations
  - Systematic parameters

$$\chi^2 = \sum_{i=1}^{n^{\exp}} \left[\sum_{j=1}^{n^{data}} \left(g_{1,j}^{data} + S_i \cdot \Delta_{sys} g_{1,j}^{data} - \frac{g_{1,j}^{theor}}{N_i} \right) Cov_{jk}^{-1} \left(g_{1,k}^{data} + S_i \cdot \Delta_{sys} g_{1,k}^{data} - \frac{g_{1,k}^{theor}}{N_i} \right) + S_i^2 + \frac{(N_i - 1)^2}{\Delta N_i^2} \right]$$

❖ Also in this case the parameters S_i and N_i can be calculated analytically at each step, but their expression is more complicated

Results Stability

- ***Up to three minima*** *have recently been seen with similar values of* χ^2.
  - ***Test the stability and accuracy*** *of the methods using* ***MonteCarlo pseudo-data*** *generated from a chosen set of polarised parton distributions*
  - ***compare then with the fit results.***



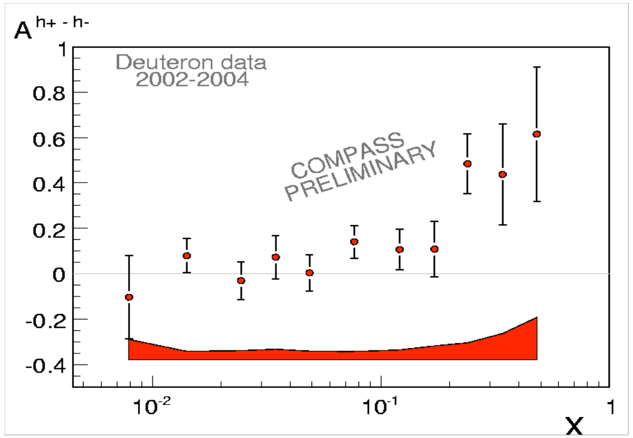
LSS'06

Symmetric Sea Assumption: new results from COMPASS

Difference asymmetry

$$A^{+-} = \frac{(\sigma^{h+}_{\uparrow\downarrow} - \sigma^{h-}_{\uparrow\downarrow}) - (\sigma^{h+}_{\uparrow\uparrow} - \sigma^{h-}_{\uparrow\uparrow})}{(\sigma^{h+}_{\uparrow\downarrow} - \sigma^{h-}_{\uparrow\downarrow}) + (\sigma^{h+}_{\uparrow\uparrow} - \sigma^{h-}_{\uparrow\uparrow})}$$

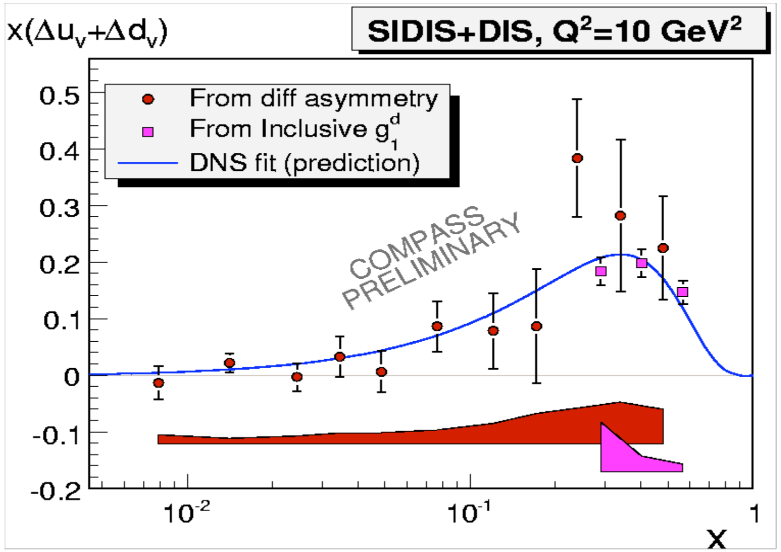
$$A_d^{\pi^+-\pi^-}(x) = A_d^{K^+-K^-}(x) = \frac{\Delta u_v(x) + \Delta d_v(x)}{u_v(x) + d_v(x)}$$

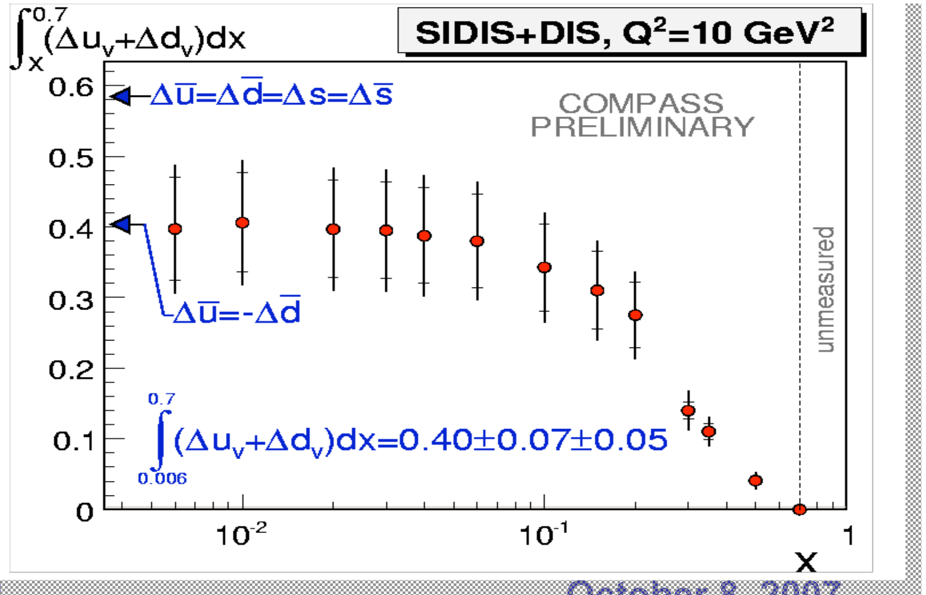


$$\Gamma_v \equiv \int_0^1 (\Delta u_v(x) + \Delta d_v(x))dx$$

$$\begin{aligned}\Delta\bar{u} + \Delta\bar{d} &= (\Delta s + \Delta\bar{s}) + \frac{1}{2}(a_8 - \Gamma_v) \\ &= 3\,\Gamma_1^N - \frac{1}{2}\Gamma_v + \frac{1}{12}a_8\end{aligned}$$

ϖ *The estimated Γ_v is 2.5σstat away from the symmetric sea scenario*





Conclusions

- *At the moment still other data* ***(like $A_{LL}^{\pi_0}$ for AAC or semi-inclusive asymmetries for De Florian et al.)*** *has to come in aid of QCD fits in order to pin down the gluon distribution*
  - ***For more precise data on ΔG from scaling violations of g_1, proposed e-p colliders e-LIC and eRHIC***

- *The latest QCD fits look at various aspects of Δq* ***(AAC:gluon, LSS:HT…)***
  - *It would be nice to have one comprehensive analysis with all these features:*
    - ***HT calculation***
    - ***Statistical error band calculation***
      - ***explicitely state which $\Delta\chi^2$ choice was made and possibly provide results with the two choices***
    - ***Propagation of systematic uncertainties***
    - ***Fit NS to test the Bjorken Sum Rule***
    - ***Fit α_s***
    - ……

For Pamela

QCD fits to the g_1 world data and uncertainties

Lara De Nardo
TRIUMF/DESY

In my talk I will mainly focus on the calculation of uncertainties in polarised parton distributions obtained by QCD fits to g_1 world data. I will discuss how some published results on statistical uncertainties cannot be readily compared with results from other papers, because of different interpretations of these uncertainties; from this it follows that sometimes opposite conclusions are reached based on similar results.

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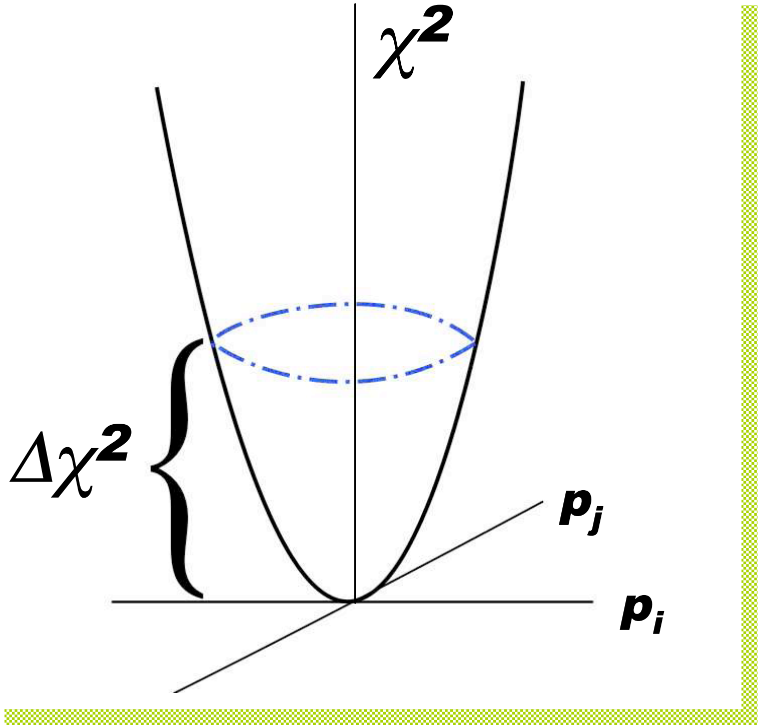

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Systematic uncertainties

One can get even more fancy and consider the experimental systematic uncertainties:

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$$\left(\frac{\partial \chi^2}{\partial S_i} = 0 \right) \Rightarrow S_i = \frac{\sum_{k=1}^{n^{data}} \left(g_{1,k}^{theor} - g_{1,k}^{data}\right) \Delta_{sys} g_{1,k}^{theor} / \left(\Delta g_{1,k}^{data}\right)^2}{1 + \sum_{j=1}^{n^{data}} \left(\Delta_{sys} g_{1,j}^{theor}\right)^2 / \left(\Delta g_{1,j}^{data}\right)^2}$$